Outline of Research

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I. Algebra

(a) Group Ring

The paper [15] dealt with a special case concerning the problem how the structure of the group ring *KG* over *K* of *G* relates to the structure of *G*, where *K* be an algebraically closed field of a finite group *G* and a prime number *p*. In particular, [15] completely determined the structure of *G* which enables the radical of *KG* to be a principal ideal. This result is introduced in the book written by C. Faith, Algebra II, 1976. F. Kasch guided his student to choose to give another proof of the result as a dissertation.

(b) Equivalence and Duality

In [28] I gave a proof to a theorem in order to prove the duality in the quasi-Frobenius ring. (It is described as the Morita-Tachikawa theorem in the book (1962) written by Curtis-Reiner.) This paper [28], together with the theorem 4.1 in [33], was the first step toward the generalization of the duality theory.

Categorical observation in ring theory was introduced for the first time in [34]. In this particular paper, a duality between two categories was defined, and the general theory of dualities was developed based on the definition. Similarly, from the viewpoint of category theory, this paper defined equivalence between categories and discussed when the category of left *A*-modules and the category of right *B*-modules are equivalent, obtaining a satisfactory result. Nowadays this is known as "Morita theory (Morita Equivalence and Morita Duality)", introduced in many books in the world. (e.g. N. Jacobson, Basic Algebra II, 1980; C. Faith, Algebra I, 1974; P. M. Cohn, Morita equivalence and duality, 1966; I. Reiner, Maximal orders, 1975; etc.)

The paper [38] discussed how modules are determined by endomorphism rings under the concept of [34].

(c) Frobenius Extensions

It was [43] and [45] that studied the Frobenius extension from the viewpoint of category theory. T. Nakayama and T. Tsuzuku defined the notion of β-Frobenius extensions of a ring B by introducing an automorphism β of B, but the notion exhibited a halfway feature, as was pointed out by themselves. In [45] I introduced a more general notion to dissolve this defect and proved the endomorphism ring theorem with a necessary and sufficient condition, so that I completed the halfway feature since the work of F. Kasch. [47] was a note for Frobenius extensions.

(d) Localization

Generalizations of localizations of commutative rings to non-commutative rings were given by P. Gabriel and L. Silver. In [49], [50], [54], [55], and [78], I showed that their theories essentially dealt with the same thing, though they were apparently different. At the same time I generalized the localization theory further as a theorem of a reflexive subcategory theory of the module category which becomes a Grothendieck category. [60] gave a comprehensive exposition of these results (except [78]) by adding some supplements.

(e) Others

As generalizations of quasi-Frobenius algebras, R. M. Thrall defined QF-1 algebras, QF-2 algebras, and QF-3 algebras. Some relationships among them are discussed in [35]. As for QF-3 rings, [34] established the structure theory for QF-3 algebras, [48] dealt with problems on dualities, and [52] discussed the quotient rings of Noetherian QF-3 rings. [44] discussed *S*-rings in the sense of F. Kasch.

II. Topology

(a) Uniform Topological Spaces

The paper [1] solved a problem of A. Weil concerning uniform topology. As a compactification of topological spaces, the Stone-Čech compactification and the Wallman compactification are famous. It was [12], [13], [14], and [16] that extended the notion of uniform topology so that all these were treated as completions of uniform topological spaces. These papers were introduced in the book (1964) by J. R. Isbell without being paid much attention. H. Poppe, however, introduced this uniform topology in his book, "Compactness in general function spaces", Berlin (1974), and thus proved an extension of Ascoli Theorem. Moreover, A. K. and E. F. Steiner wrote their paper entitled "On semi-uniformity", Fund. Math. 83 (1973) to interpret Morita's theory from a new point of view.

The paper [17] discussed the Endentheorie established by H. Freudenthal from the standpoint of uniform topological spaces.

(b) Dimension Theory

Before 1950 dimension theory was discussed mainly in separable metric spaces. Meanwhile [10], [11], and [26] discussed dimension theory for normal spaces. [21] and [23] established a fundamental theory of the dimension of general metric spaces (which were not necessarily separable). (This was independently discovered by M. Katětov except results concerning closed mappings.) Nowadays, most books of the dimension theory contain this theory. (A. R. Pears, "Dimension theory of general spaces", 1975; R. Engelking, "Dimension Theory", 1978; etc.)

The paper [19] discussed product theorems of dimension. Later the results were extended to more general spaces by [62], [70], and [80]. These and other various theories concerning the dimension of general topological spaces were summarized in [83].

(c) Paracompact Spaces and Product Spaces

Because of the hard times after the World War II, [8] was written without knowing the paper by Dieudonné. The paper discussed strong paracompact spaces, and is frequently quoted in research papers and books.

[37] was written at the request of C. Kuratowski, on the occasion of

publication of the memorial 50th volume of Fundamenta Mathematicae. The main theorem of this paper was to characterize a topological space X so that $X \times I^m$ (I=[0, 1], where m is an infinite cardinal number.) be normal, and it characterized paracompactness of X when m was not less than the weight of X. ([36] showed that similar results could be obtained even if we took the two point subspace $\{0, 1\}$ instead of I.)

[20] proved the paracompactness of *CW*-complexes for the first time. (Although some books state that H. Miyazaki (Tohoku Mathematical Journal (1952)) was the first to give the proof, it is not true. A group of topologists in Osaka pointed out that his proof contained a decisive gap.) Defining a hereditary weak topology, my proof given to the general theorem concerning this in [22] was complicated by use of induction, but in [37], another proof with better perspective was given.

The papers [39], [40], [41], and [42] discussed normality of product spaces. Particularly in [39], defining a *P*-space, it was proved that a topological space whose product with every metric space is normal is a normal *P*-space. Moreover in [40], *M*-space was defined as an example of *P*-space. This extended both the compact spaces and the metric spaces simultaneously, and was later found to coincide with the p-space of A. V. Arhangel'skii in case that the space is paracompact, which led to inventions of various generalized metric spaces.

(d) Metrizability

Metrizability is an old and new problem. The two metrization theorems given in [16] and [23] were among the basic ones. I showed in [24] condition under which metrizability is preserved by closed mappings. Because the same was independently obtained by A. H. Stone, it is called the Hanai-Morita-Stone Theorem.

(e) Cohomology Groups and Cohomotopy Groups

The paper [18] extended the cohomotopy group theory to the paracompact spaces, while [70] discussed Čech cohomology groups for general topological spaces. [74], which was written at the request on the occasion of the foundation of Houston J. Math., extended the H. Hopf type extension theorem, which was obtained for normal spaces in [2], to general topological spaces.

(f) Shape Theory

In 1968, K. Borsuk introduced the notion of shape as an extension of homotopy for the compact metric spaces. This notion has been extended to metric spaces by R. H. Fox, to compact Housdorff spaces by S. Mardešić and J. Segal, and finally to general topological spaces by S. Mardešić. In [69] I developed their results by viewing the shape theory as the pro-homotopy category theory for *CW*-complexes. It was in [67] that a then pending problem was proved in the finite-dimensional case completely in the same form as the case of complex, (The same does not hold in the infinite-dimensional case.) The recently published S. Mardešić book "Shape Theory" (1981) is based on my method.

The papers [65], [66], [71], [73], [75], [76], and [77] were all related to the shape theory.

(g) Others

The paper [72] proved the most general extension of the Borsuk's homotopy extension theorem, and [68] made its essence clear.

On the other hand, [25] was to characterize the image space of the unit open interval under a perfect mapping, as an analogue (and an extension) of the Hahn-Mazurkiewicz theorem which characterized the continuous image of the unit closed interval.

III. Others

The papers [3], [4], and [6], which were motivated by M. Sugawara’s research, mainly dealt with the determination of group of analytical automorphisms, an analogue of the Schwarzian lemma, and so on, for the three types of irreducible symmetric bounded domains. In the [27], it was discussed to deal with those for the remaining types of irreducible symmetric bounded domains, to obtain the S. Bergman’s kernel function for each of these four types of domains, and to give the Poisson’s and the Cauchy’s integral representation by using the Šilov boundary.

The [5] systematically discussed the problem concerning deformation of symmetric matrix and skew-symmetric matrix of complex coefficients by unitary matrix, from the viewpoint of the anti-linear transformation, which emerged during the above mentioned studies. The paper [7] gave a generalization of the results to operators of Hilbert space.