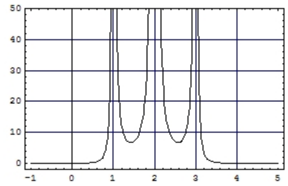
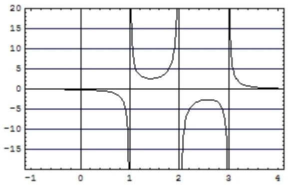
5. *Man skizziere die Funktionen der Form* $(x-1)^{n_1}(x-2)^{n_2}(x-3)^{n_3}$ *mit* $n_i \in \{-2,-1,1,2\}$

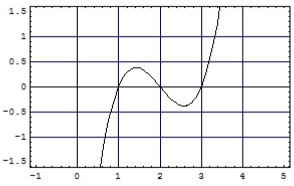
$(x-1)^{-2}(x-2)^{-2}(x-3)^{-2}$



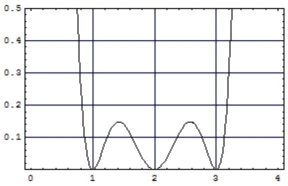
$(x-1)^{-1}(x-2)^{-1}(x-3)^{-1}$



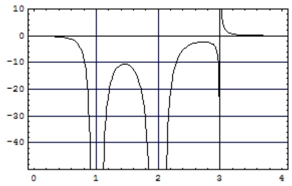
$(x-1)^{1}(x-2)^{1}(x-3)^{1}$



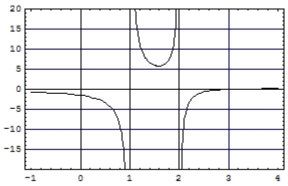
$(x-1)^{2}(x-2)^{2}(x-3)^{2}$



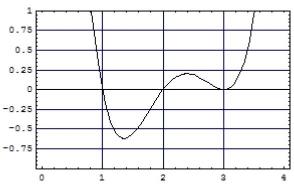
$(x-1)^{-2}(x-2)^{-2}(x-3)^{-1}$



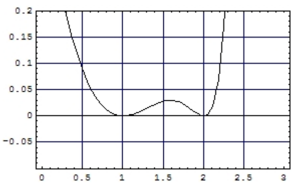
$(x-1)^{-1}(x-2)^{-1}(x-3)^{1}$



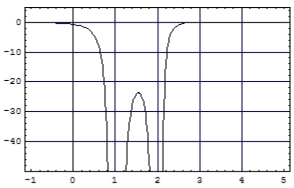
$(x-1)^{1}(x-2)^{1}(x-3)^{2}$



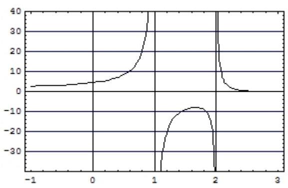
$(x-1)^{2}(x-2)^{2}(x-3)^{-2}$



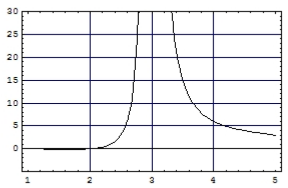
$(x-1)^{-2}(x-2)^{-2}(x-3)^{1}$



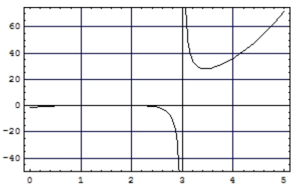
$(x-1)^{-1}(x-2)^{-1}(x-3)^{2}$



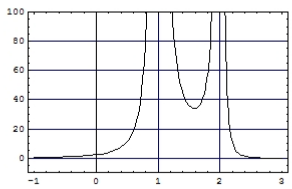
$(x-1)^{1}(x-2)^{1}(x-3)^{-2}$



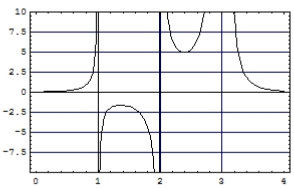
$(x-1)^{2}(x-2)^{2}(x-3)^{-1}$



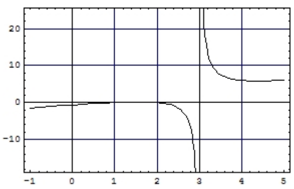
$(x-1)^{-2}(x-2)^{-2}(x-3)^{2}$



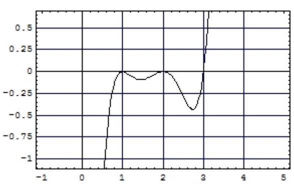
$(x-1)^{-1}(x-2)^{-1}(x-3)^{-2}$



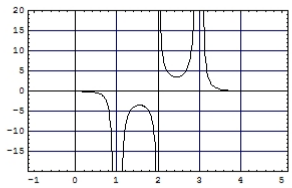
$(x-1)^{1}(x-2)^{1}(x-3)^{-1}$



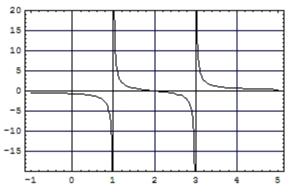
$(x-1)^{2}(x-2)^{2}(x-3)^{1}$



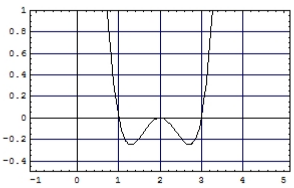
$(x-1)^{-2}(x-2)^{-1}(x-3)^{-2}$



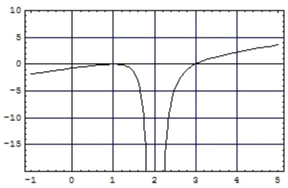
$(x-1)^{-1}(x-2)^{1}(x-3)^{-1}$



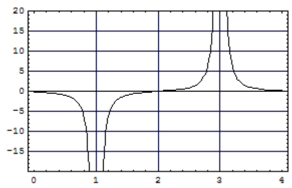
$(x-1)^{1}(x-2)^{2}(x-3)^{1}$



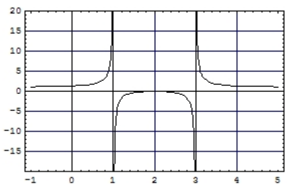
$(x-1)^{2}(x-2)^{-2}(x-3)^{2}$



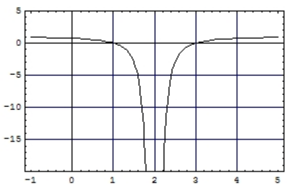
$(x-1)^{-2}(x-2)^{1}(x-3)^{-2}$



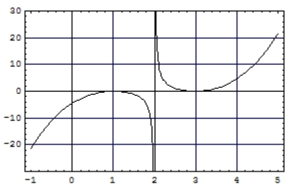
$(x-1)^{-1}(x-2)^{2}(x-3)^{-1}$



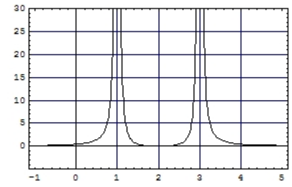
$(x-1)^{1}(x-2)^{-2}(x-3)^{1}$



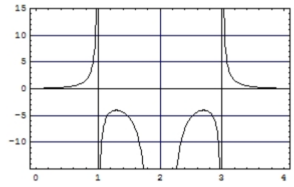
$(x-1)^{2}(x-2)^{-1}(x-3)^{2}$



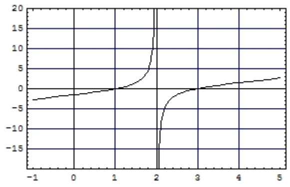
$(x-1)^{-2}(x-2)^{2}(x-3)^{-2}$



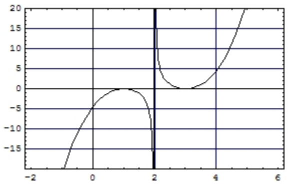
$(x-1)^{-1}(x-2)^{-2}(x-3)^{-1}$



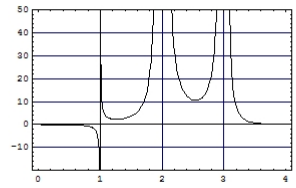
$(x-1)^{1}(x-2)^{-1}(x-3)^{1}$



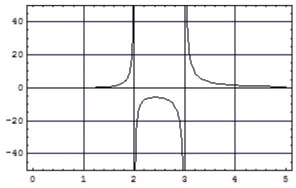
$(x-1)^{2}(x-2)^{-1}(x-3)^{2}$



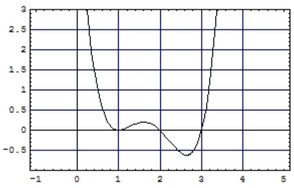
$(x-1)^{-1}(x-2)^{-2}(x-3)^{-2}$



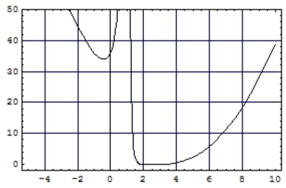
$(x-1)^{1}(x-2)^{-1}(x-3)^{-1}$



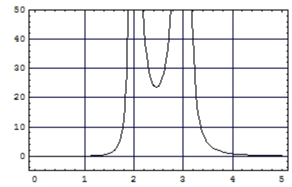
$(x-1)^{2}(x-2)^{1}(x-3)^{1}$



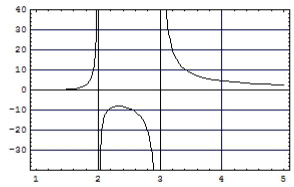
$(x-1)^{-2}(x-2)^{2}(x-3)^{2}$



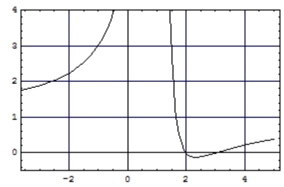
$(x-1)^{1}(x-2)^{-2}(x-3)^{-2}$



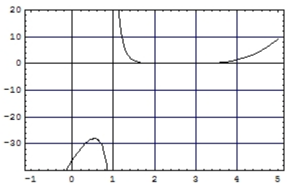
$(x-1)^{2}(x-2)^{-1}(x-3)^{-1}$



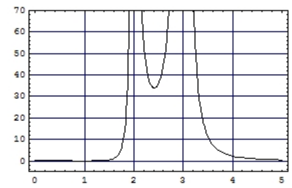
$(x-1)^{-2}(x-2)^{1}(x-3)^{1}$



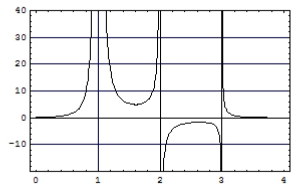
$(x-1)^{-1}(x-2)^{2}(x-3)^{2}$



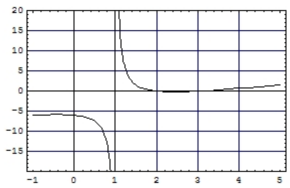
$(x-1)^{2}(x-2)^{-2}(x-3)^{-2}$



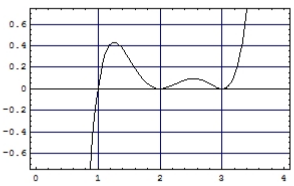
$(x-1)^{-2}(x-2)^{-1}(x-3)^{-1}$



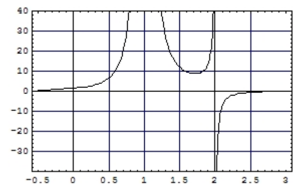
$(x-1)^{-1}(x-2)^{1}(x-3)^{1}$



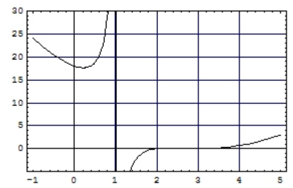
$(x-1)^{1}(x-2)^{2}(x-3)^{2}$



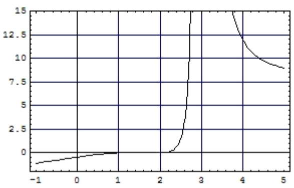
$(x-1)^{-2}(x-2)^{-1}(x-3)^{1}$



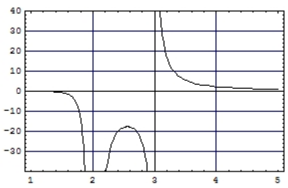
$(x-1)^{-1}(x-2)^{1}(x-3)^{2}$



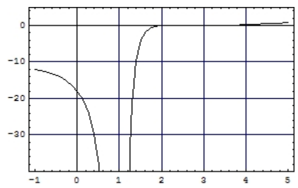
$(x-1)^{1}(x-2)^{2}(x-3)^{-2}$



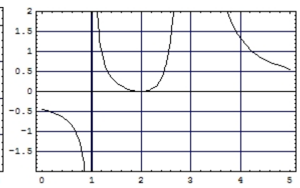
$(x-1)^{2}(x-2)^{-2}(x-3)^{-1}$



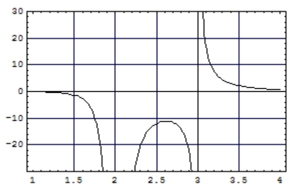
$(x-1)^{-2}(x-2)^{1}(x-3)^{2}$



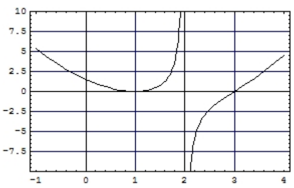
$(x-1)^{-1}(x-2)^{2}(x-3)^{-2}$

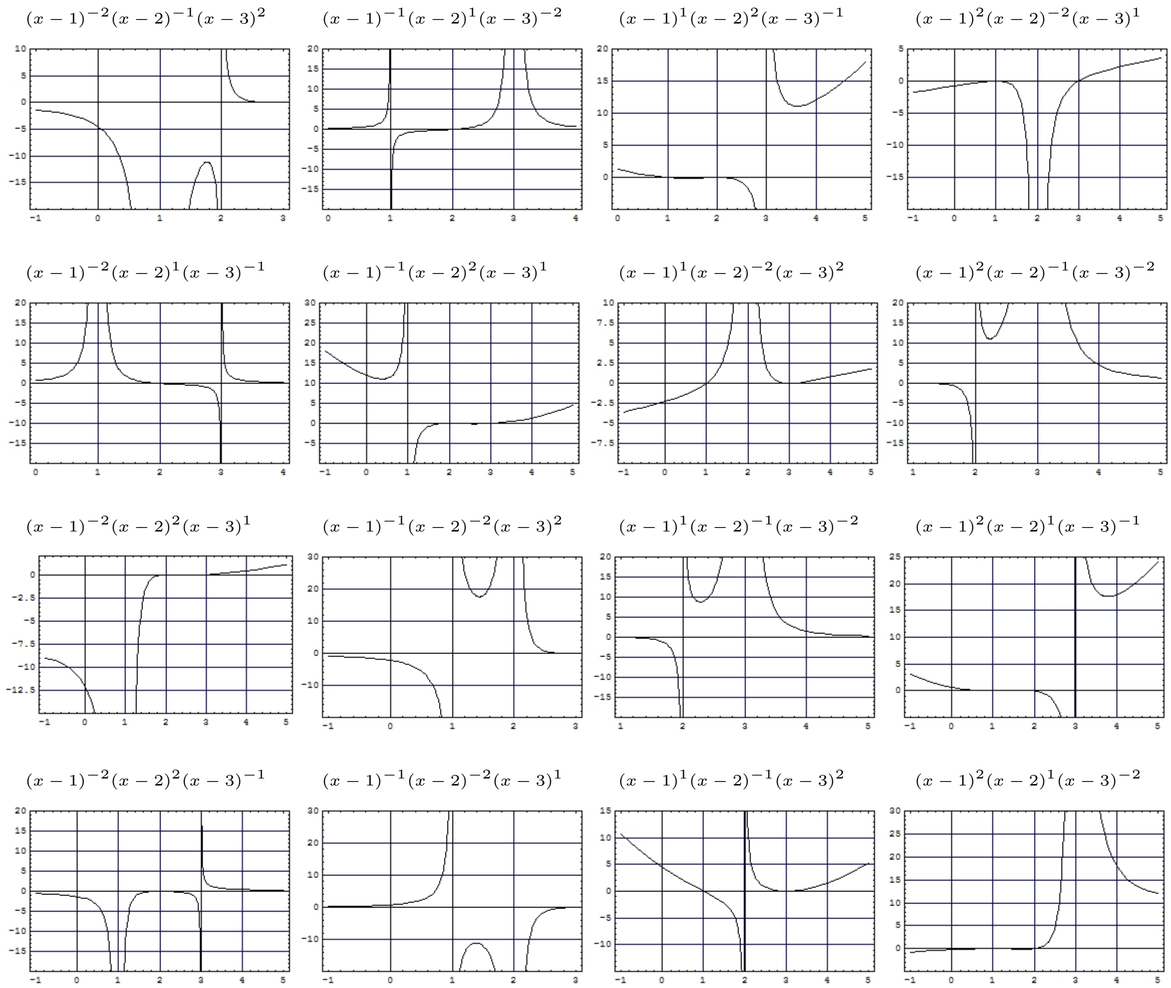


$(x-1)^{1}(x-2)^{-2}(x-3)^{-1}$



$(x-1)^{2}(x-2)^{-1}(x-3)^{1}$

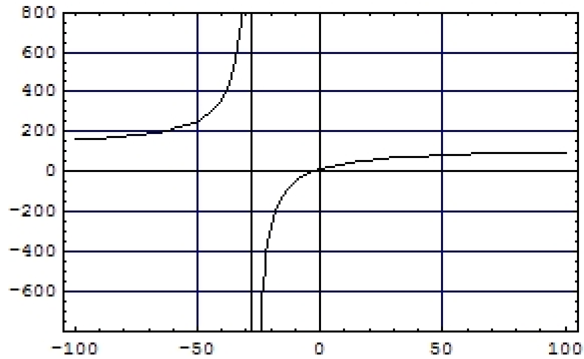




6. Wenigstens eine Anwendungs-Aufgabe. *Wir betrachten die Fisch-Population $L(t)$ eines Sees. Die Anfangspopulation (Zeitpunkt $t = 0$) bestehe aus 10 Fischen, langfristig stabilisiere sie sich bei 110 Fischen. Die anfängliche Wachstumsrate sei $L'(0) = 4$. Man gehe davon aus, dass die Entwicklung durch eine Michaelis-Menton-Funktion $L(x) = \dfrac{B \cdot x}{x+K} + L_0$ mit $B, K, L_0 > 0$ beschrieben wird.*
Bestimmen Sie den Zeitpunkt, zu dem sich die Anfangspopulation verfünffacht hat. Zeichnen Sie die Funktion in einem Koordinatensystem.

- Die Anfangspopulation besteht aus 10 Fischen, d.h. $L(0) = L_0 = 10$.
- Die Wachstumschranke ist bei 110 Fischen, d.h. $B = 110$.
- Die anfängliche Wachstumsrate ist $L'(0) = 4$. Aus $L'(x) = \dfrac{110 \cdot K}{(x+K)^2}$ erhält man $L'(0) = \dfrac{110 \cdot K}{K^2} = 4$, d.h. $K = 27,5$.

Die Michaelis-Menton-Funktion für diese Situation ist $L(x) = \dfrac{110 \cdot x}{x + 27,5} + 10$.



Gesucht ist also ein x_0 mit $L(x_0) = 50$:

$$\begin{aligned}
\frac{110 \cdot x_0}{x_0 + 27,5} + 10 &= 50 \\
&\Downarrow \\
\frac{110 \cdot x_0}{x_0 + 27,5} &= 40 \\
&\Downarrow \\
110 \cdot x_0 &= 50\,(x_0 + 27,5) = 50x_0 + 1375 \\
&\Downarrow \\
60x_0 &= 1375 \\
&\Downarrow \\
x_0 &= 22,9166
\end{aligned}$$