Übungsaufgaben 5.

1. We have considered the following classes of finite-dimensional indecomposable Kronecker modules:

$$\begin{aligned}\mathcal{R}_0 &= \{M \mid \ M_\alpha \text{ is invertible and } M_\beta M_\alpha^{-1} \text{ is nilpotent}\}\\ \mathcal{B} &= \{M \mid \ \text{both } M_\alpha, M_\beta \text{ are invertible}\}\\ \mathcal{R}_\infty &= \{M \mid \ M_\beta \text{ is invertible and } M_\alpha M_\beta^{-1} \text{ is nilpotent}\}\end{aligned}$$

Let M, N be finite-dimensional indecomposable Kronecker modules which are neither both in $\mathcal{R}_0$, nor both in $\mathcal{B}$, nor both in $\mathcal{R}_\infty$. Show that $\mathrm{Hom}(M, N) = 0$.

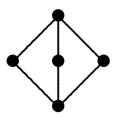
2. Let V and W be S-modules. Then the product set $V \times W$ is again an S-module which is the direct sum of $V = V \times 0$ and $W = 0 \times W$ (thus, often we write just $V \oplus W$ instead of $V \times W$). Let $f\colon V \to W$ be a module homomorphism, and let

$$\Gamma_f = \{(v, f(v)) \mid v \in V\}$$

be the *graph of f*.

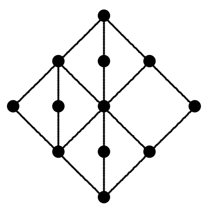
Show: The map $f \mapsto \Gamma_f$ defines a bijection between $\mathrm{Hom}(V, W)$ and the set of submodules $U \subseteq V \times W$ with $U \oplus (0 \times W) = V \times W$.

3. (a) Show: If M is a module with submodule lattice



then the base field k is the field with two elements.

(b) Show: The following lattice cannot be the submodule-lattice of a module:



4. Let V be a module of finite length, and let $V_1, \dots, V_t$ be submodules of V with $\sum_i V_i = V$. Show: If

$$\sum_{i=1}^{t} l(V_i) = l(V),$$

then $V = \bigoplus_{i=1}^{t} V_i$.