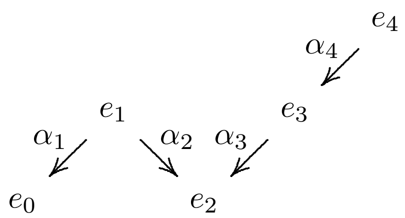
Übungsaufgaben 8.

Definitions: Let M be a module of finite length. Then M is said to be *serial* provided it has a unique composition series. Also, M is said to be *local* provided it has a unique maximal submodule (a *maximal* submodule is a proper submodule which contains any proper submodule).

1. Show that any local module is indecomposable.

Now let $Q = (Q_0, Q_1, Q_2)$ be a complete gentle quiver.

2. Let $\alpha \in Q_1$, consider the word $w = (\alpha_1; +, -, +, +) \in \Omega(\alpha)$. Let $M(w)$ be the corresponding string module. To fix the notation, let $(\alpha_1; +, -, +, +) = \alpha_1\alpha_2^{-1}\alpha_3\alpha_4$ with arrows $\alpha_i \in Q_1$, and let $M(w)$ be the vector space with basis e_0, e_1, e_2, e_3, e_4, such that the module structure is given as indicated by the following picture:



(a) Show that $M(w)$ has at least 9 composition series.

(b) Show: In case $|Q_0| = 1$, the module $M(w)$ has more than 9 composition series.

3. Local string modules. Let $\alpha \in Q_1$, consider the word $w = (\alpha_1; \epsilon_1, \dots, \epsilon_n) \in \Omega(\alpha)$, where all $\epsilon_i \in \{+, -\}$. (a) Show that the string module $M(w)$ is local if and only if there is some $0 \le t \le n$ such that $\epsilon_i = +$ for $1 \le i \le t$ and $\epsilon_i = -$ for $t+1 \le i \le n$. (b) When is $M(w)$ serial?

4. Local band modules. Let $w \in \Omega(\alpha)$ be a primitive cyclic word and ϕ an automorphism of the vector space V. Let $M(w, \phi)$ be the corresponding band module. Show: If the band module $M(w, \phi)$ is local, then it is a factor module $M(v)/U$ of a local string module $M(v)$, where U is a one-dimensional submodule.