**Übungsaufgaben 9.**

**1.** Construct a simple 2-module of dimension $n$, for any $n \in \mathbb{N}_1$, and also one which is infinite-dimensional.

**2.** Construct, if possible, a $2 \times 2$-matrix $B$ with coefficients in $\mathbb{C}$ such that $(\mathbb{C}^2; A, B)$ is simple, for $A$ as follows:

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

**3.** Let $Q$ be a finite quiver with $n$ vertices. Show that the number of simple representations is $n$ if and only if there are no oriented cycles.

**4.** (a) Let $k$ be a field. Show that $(k[T], T\cdot)$ has no simple submodules.

(b) A module $M$ is called *uniform* (or *colocal*) provided it contains a non-zero submodule which is contained in any non-zero submodule. Of course, a uniform module has a unique simple submodule and is indecomposable. Construct a 1-module with a unique simple submodule which is not uniform. Construct a 2-module with a unique simple submodule which is not uniform, but is in addition also indecomposable.