

# Shimura Varieties

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WS 2021/2022

Thursday, 10–12, starting 21/10/2021, in Room U5-133

Numbers in parentheses represent sections in Milne's "Introduction to Shimura Varieties" (available at <https://www.jmilne.org/math/xnotes/svi.pdf>)

- 1. Hermitian Symmetric Domains (1): Simon** Modular curves are quotients of the upper half plane by a group action. For Shimura varieties, the upper half plane has to be replaced by a more general notion, namely Hermitian symmetric domains.
- 2. Hodge Structures and Their Classifying Spaces (2): Johannes** Hermitian symmetric domains form parameter spaces for Hodge structures.
- 3. Locally Symmetric Varieties (3): Karsten** We consider quotients of Hermitian symmetric domains by discrete groups and give them the structure of an algebraic variety.
- 4. Connected Shimura Data and Connected Shimura Varieties (4): Sören** A preliminary stage of the definition of a Shimura variety. Locally symmetric varieties where the discrete group is obtained by congruence relations.
- 5. Shimura Data and Shimura Varieties (5): Sarah** The full definition. Uses reductive groups instead of semisimple groups.
- 6. The Siegel Modular Variety and Shimura Varieties of Hodge Type (6, 7): Lars** The Siegel modular variety is the most basic example of a Shimura variety. It is further generalised by Shimura varieties of Hodge type.
- 7. PEL Shimura Varieties (8): Simon** Construction of Shimura varieties that classify Abelian varieties with a polarisation, endomorphism, and level structure.
- 8. General Shimura Varieties (9): Alex** More Shimura varieties can be realised as classifying space by the use of Abelian motives.
- 9. Complex Multiplication (10, 11): Sören** Shimura varieties can always be defined over a number field  $E$ , so the group  $\text{Aut}(\mathbb{C}/E)$  acts on it. The theory of complex multiplication helps to describe this action on special points.
- 10. Canonical Models (12, 13): TBA** A Shimura variety admits several models over number fields. Such a model is canonical if it satisfies a reciprocity law at the special points. Canonical models are unique.
- 11. Existence of Canonical Models (14): TBA** Sketch of the construction of the canonical model of a Shimura variety.