

Fibonacci-Zahlen

Definition. Die Fibonacci-Zahlen sind die Folge von Zahlen $(F_i)_{i \in \mathbb{N}}$ definiert durch

1. $F_0 = 0, F_1 = 1,$
2. $F_i = F_{i-1} + F_{i-2}$ für alle $i \in \mathbb{N}, i \geq 2.$

Fibonacci-Zahlen

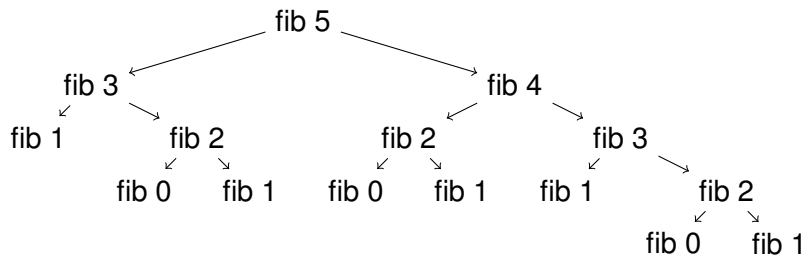
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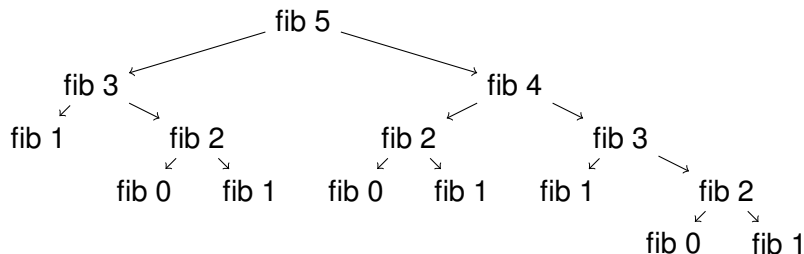
Berechnung:

```
fib :: Int -> Int
fib 0 = 0
fib 1 = 1
fib n = (fib (n - 1)) + (fib (n - 2))
```

Fibonacci-Zahlen: Rekursion



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... F_{200} ?! Effizienter durch Memoization.

Fibonacci-Zahlen: Geschlossene Form

Sei $\varphi = \frac{\sqrt{5}+1}{2} \approx 1,618$ der *goldene Schnitt*.

Satz (de Moivre – Binet). $F_n = \frac{\varphi^n - (-\varphi)^{-n}}{\varphi - (-\varphi)^{-1}} = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}} = \left\lfloor \frac{\varphi^n}{\sqrt{5}} \right\rfloor$

```
cfib :: Int -> Integer
cfib n = (round (phi^n / sqrtfive))
  where sqrtfive = (sqrt 5.0)
        phi = ((1 + sqrtfive) / 2)
```

$F_{200} = 280571172992509965361722520092440986124288$

Korollar. $\lim_{n \rightarrow \infty} F_{n+1}/F_n = \varphi$

55

34

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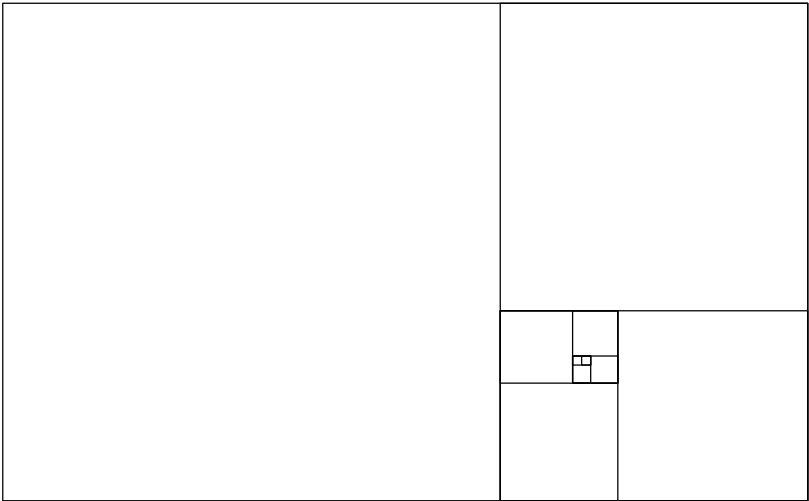
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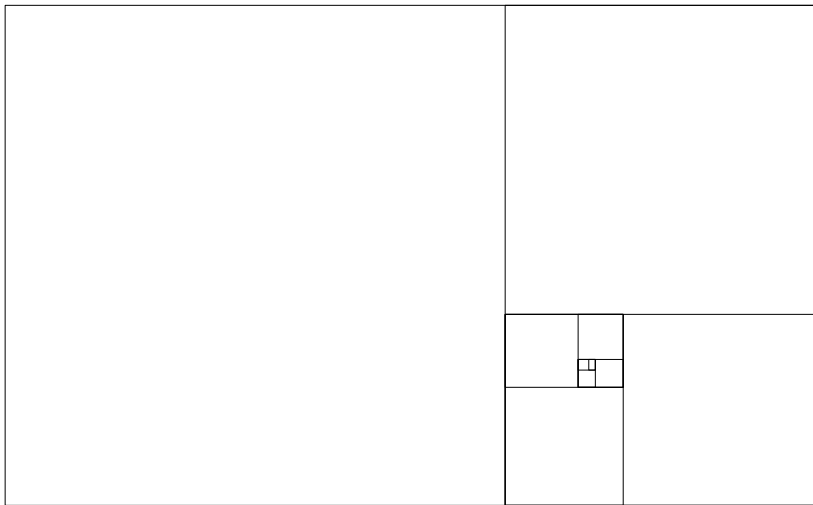
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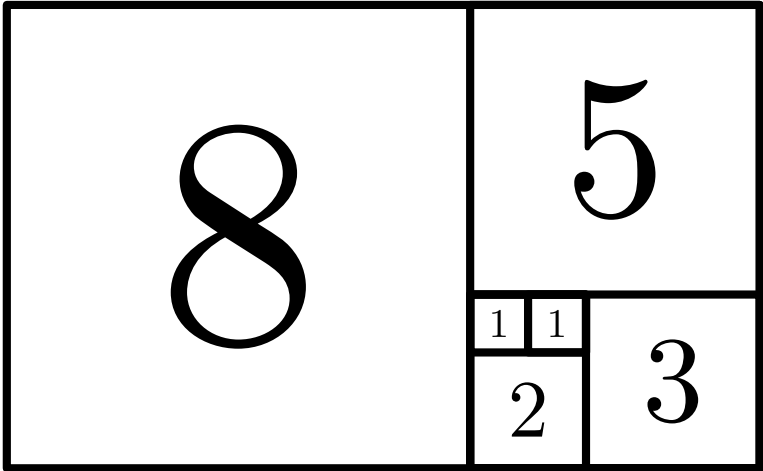
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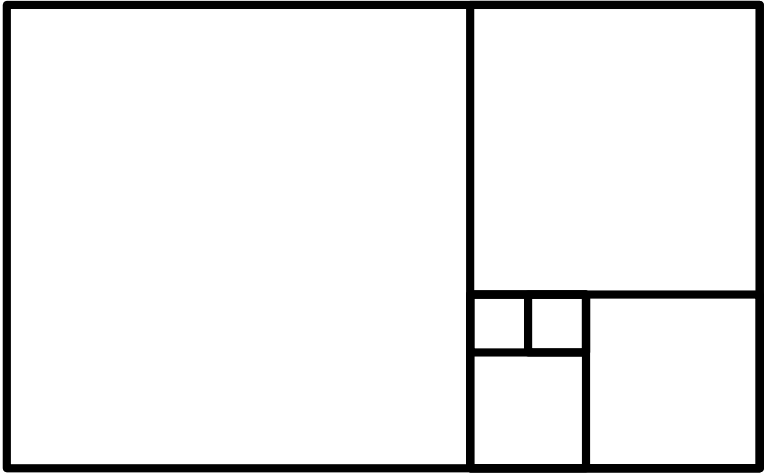
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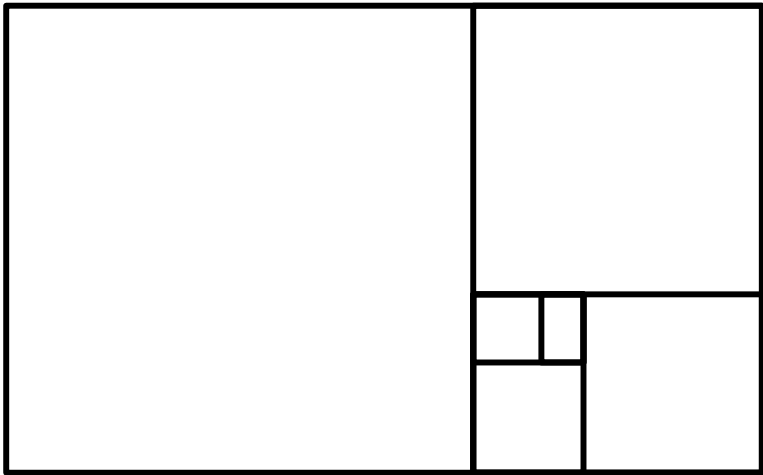
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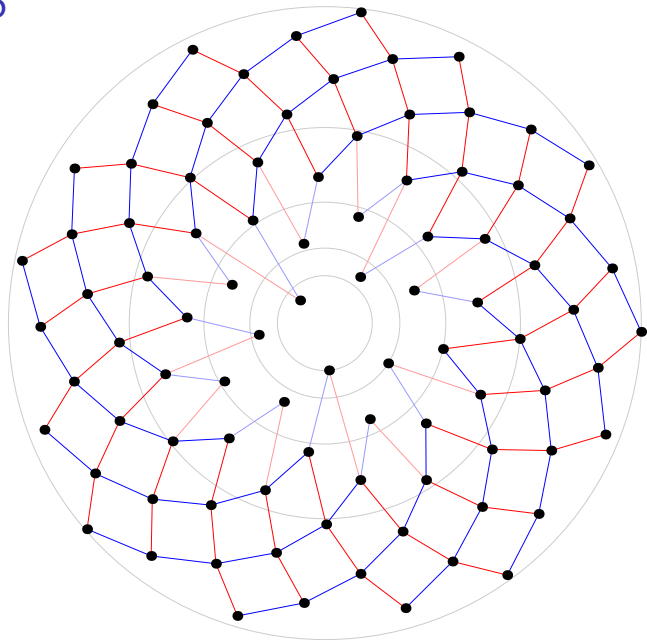




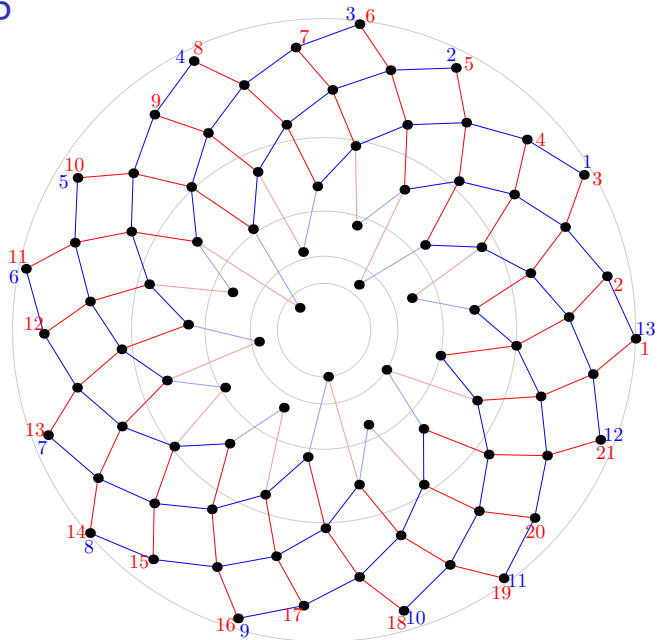
Sonnenblumen



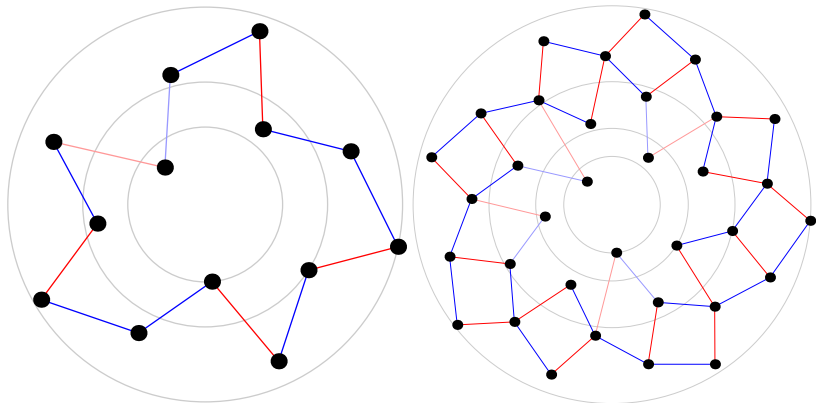
Größe 5



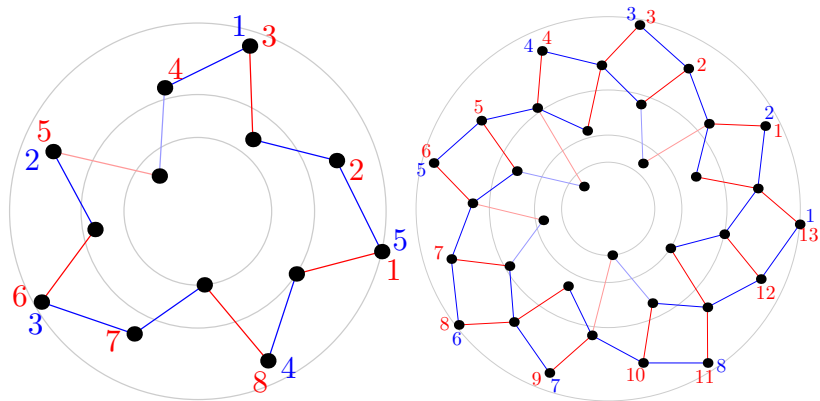
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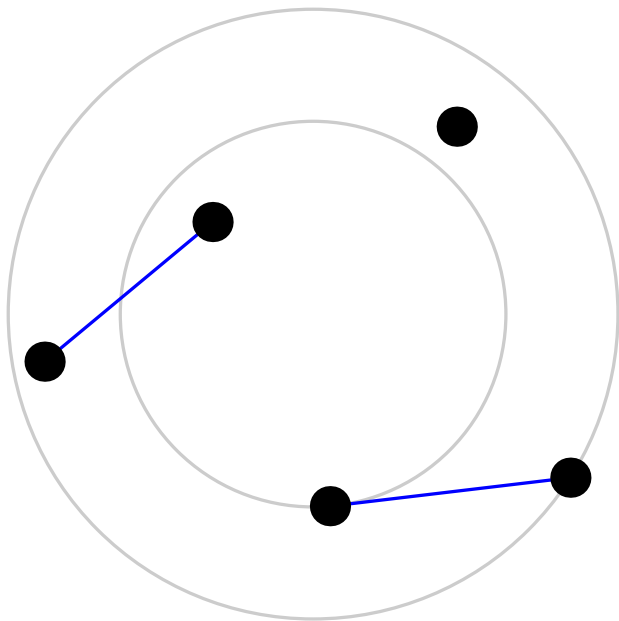
Größe 3, 4



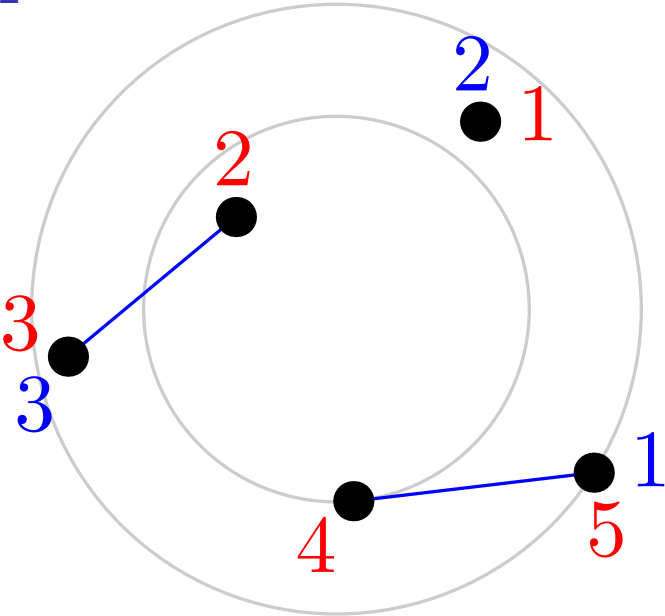
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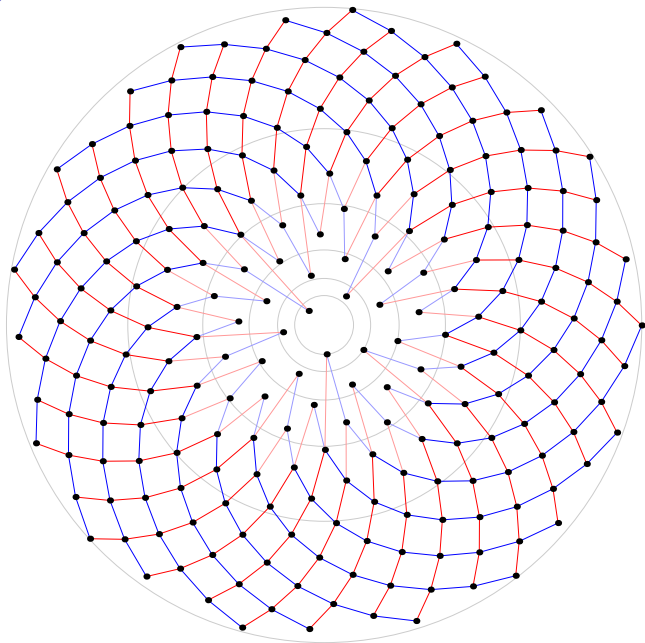
Größe 2



Größe 2



Größe 6



Größe 6

