

Limit Theorems for Linear Eigenvalue Statistics of Matrix Models

L. Pastur

Institute for Low Temperature Physics,
Kharkiv, Ukraine

Abstract

We study the variance and the characteristic functional of linear eigenvalue statistics of unitary invariant matrix models of $n \times n$ Hermitian matrices as $n \rightarrow \infty$. Assuming that the test function of the statistic is smooth enough and using the asymptotic formulas by Deift et al for orthogonal polynomials with varying weight, we show first that if the support of the density of states of the model consists of $q \geq 2$ intervals, then in the global regime the variance is a quasi-periodic function of n as $n \rightarrow \infty$ generically in the potential, determining the model. We show next that the exponent of the characteristic functional in general is not $1/2 \times$ the variance, as it should be if the Central Limit Theorem would be valid, and we find the asymptotic form of the characteristic functional in certain cases. We also show that in the intermediate and the local regimes the Central Limit Theorem is valid.

References

- [1] S. Albeverio, L. Pastur, M. Shcherbina, *On the $1/n$ -expansion for some unitary invariant ensembles of random matrices*. Commun. Math. Phys. **222** (2001) 271 - 305
- [2] J. Ambjørn, J. Jurkiewicz, Yu. Makeenko, *Multiloop correlators for two-dimensional quantum gravity*, Phys. Lett. **B251** (1990) 517-524
- [3] E. Basor, *Distribution functions for random variables for ensembles of positive Hermitian matrices*. Comm. Math. Phys. **188** (1997) 327-350
- [4] G. Bonnet, F. David, B. Eynard, *Breakdown of universality in multi-cut matrix models*, J. Phys. **A33** (2000) 6739-6768
- [5] A. Boutet de Monvel, L. Pastur, M. Shcherbina, *On the statistical mechanics approach to the random matrix theory: the integrated density of states*, J. Stat. Phys. **79** (1995) 585-611
- [6] E. Brézin, A. Zee, *Universality of the correlations between eigenvalues of large random matrices*, Nuclear Phys. **B402** (1993) 613-627

- [7] V. S. Buyarov, E. A. Rakhmanov, *Families of equilibrium measures in an external field on the real axis*, Sb. Math. **190** (1999) 791-802
- [8] T. Cabanal-Duvillard, *Fluctuations de la loi empirique de grandes matrices aléatoires*, Ann. Inst. H. Poincaré Probab. Statist. **37** (2001) 373-402
- [9] O. Costin, J.L. Lebowitz, *Gaussian fluctuations in random matrices*. Phys. Rev. Lett. **75** (1995) 69-72
- [10] P. Deift, T. Kriecherbauer, K. McLaughlin, S. Venakides, X. Zhou, *Uniform asymptotics for polynomials orthogonal with respect to varying exponential weights and applications to universality questions in random matrix theory*, Comm. Pure Appl. Math. **52** (1999) 1335-1425
- [11] P. Diaconis, S. Evans, *Linear functionals of eigenvalues of random matrices*. Trans. of AMS **353**, 2615-2633 (2001)
- [12] V. Girko, *Theory of Stochastic Canonical Equations*, vols.I, II Kluwer, Dordrecht, 2001
- [13] K. Johansson, *On fluctuations of eigenvalues of random Hermitian matrices*, Duke Math. J. **91** (1998) 151-204
- [14] B. Khoruzhenko, A. Khorunzhy, L. Pastur, *1/n-corrections to the Green functions of random matrices with independent entries*, J. of Physics A: Math. and General, **28** (1995) L31-L35
- [15] A. B. J. Kuijlaars, K. McLaughlin, *Generic behaviour of the density of states in random matrix theory and equilibrium problem of real analytic external field*, Commun. Pure Appl. Math. **53** (2000) 736-785
- [16] L. Pastur, *Spectral and probabilistic aspects of matrix models*, In: *Algebraic and Geometric Methods in Mathematical Physics*, A. Boutet de Monvel and V. Marchenko (Eds.), Kluwer, Dordrecht, 1996, pp. 207-242
- [17] L. Pastur, *Random matrices as paradigm*, In: *Mathematical Physics 2000*, A. Grigoryan, T. Kibble and B. Zegarlinski (Eds.) Imperial College Press, London, 2000, pp. 216-266
- [18] L. Pastur, M. Shcherbina *Universality of the local eigenvalue statistics for a class of unitary invariant matrix ensembles*, J. Stat. Phys. **86** (1997) 109-147
- [19] Ya. Sinai, A. Soshnikov, *Central limit theorem for traces of large random symmetric matrices with independent matrix elements*. Bol. Soc. Brasil. Mat. (N.S.) **29** (1998) 1-24
- [20] A. Soshnikov, *The central limit theorem for local linear statistics in classical compact groups and related combinatorial identities*. Ann. Probab. **28**, 1353-1370 (2000)

- [21] A. Soshnikov, *Gaussian limit for determinantal random point fields*. Ann. Probab. 30 (2002) 171–187
- [22] H. Spohn, *Interacting Brownian particles: A study of Dyson's model*. In Hydrodynamic Behavior and Interacting Particle Systems (G. Papanicolau, ed.). Springer, New York., 1987
- [23] K. Wieand, *Eigenvalue distribution of random unitary matrices*, Prob. Theory Relat. Fields, **123** (2002) 202–224