

EXPONENTIAL FAMILIES AND UNIVERSALITY IN FIXED TRACE MATRIX ENSEMBLES

FRIEDRICH GÖTZE AND MIKHAIL GORDIN

A (linear) unitary invariant Hermitian matrix ensemble is given by the probability measure

$$\mu_{V,N}(dM) = C_N^{-1}(V) \exp(\operatorname{tr} V(M)) \lambda(dM),$$

where V is a suitable real valued function, M runs over the set of all $N \times N$ Hermitian matrix, λ is the Lebesgue measure on the space of such matrices and $C_N(V)$ is the normalizing constant.

Fixed trace random matrix ensembles can be defined by conditioning $\mu_{V,N}$ to the sets of the form $\{M : \operatorname{tr} V(M) = c\}$.

As $N \rightarrow \infty$ and the above constant c varies along with N in an appropriate way, some results of global character on the asymptotics of spectral distribution for certain fixed trace ensembles are well known. The proof can be based on the law of large numbers (or, in physical terms, on equivalence of the canonical and the microcanonical ensembles).

However, the study of local behavior of spectral correlation functions requires more elaborated tools. We propose an approach suggested by the theory of sufficient statistics and exponential families to employ the Fourier analysis and known results on unitary invariant ensembles in the problem of universal asymptotics of correlation functions for fixed trace ensembles. As an application, we establish the universal limit of correlation functions in the bulk of the spectra for fixed trace ensembles with $V(x) = x^2/2$.