

Präsenzübungen zu Vertiefung Elementare Zahlentheorie

WS 2010/2011, Blatt 1

Exercise 1. For the following pairs of integers (a, b) , determine the greatest common divisor and give a linear representation of the form $\gcd(a, b) = xa + yb$:

$$(a) \quad (949, 559), \quad (b) \quad (3801, 1113).$$

Exercise 2. Let the sequence $(x_i)_{i \geq 0}$ be defined recursively by

$$x_0 = x_1 = 1, \quad x_i = x_{i-1} + x_{i-2} \quad \text{for } i \geq 2.$$

Prove that x_i and x_{i-1} are relatively prime for all $i \geq 1$.

Exercise 3. Let n be an integer ≥ 2 and m an integer with $n! + 2 \leq m \leq n! + n$. Prove that m is not prime.

Exercise 4. Let p be a prime number and i an integer with $0 < i < p$. Show that p divides the binomial coefficient $\binom{p}{i}$.