

# Übungen zu Vertiefung Elementare Zahlentheorie

WS 2010/2011, Blatt 4

**Exercise 13.** Let  $a$  be an integer  $> 0$  with decimal representation

$$a = a_0 + a_1 \cdot 10 + a_2 \cdot 10^2 + \dots + a_k \cdot 10^k \quad (0 \leq a_i \leq 9).$$

Show that

$$a \equiv a_0 - a_1 + a_2 - \dots + (-1)^k a_k \pmod{11}.$$

**Exercise 14.** Show for an integer  $a$ :

- (a)  $a \equiv 0 \pmod{2} \implies a^2 \equiv 0 \pmod{4}$ .
- (b)  $a \equiv 1 \pmod{2} \implies a^2 \equiv 1 \pmod{8}$ .

**Exercise 15.** Show:

- (a) If  $p$  is a prime number  $\neq 2$ , then  $p \equiv 1$  or  $3 \pmod{4}$ .
- (b) If  $p$  is a prime number  $\neq 2, 3$ , then  $p \equiv 1$  or  $5 \pmod{6}$ .

**Exercise 16.** Determine all solutions of the following congruences:

- (a)  $2x \equiv 1 \pmod{19}$ ;
- (b)  $3x \equiv 1 \pmod{19}$ ;
- (c)  $4x \equiv 6 \pmod{18}$ ;
- (d)  $20x \equiv 984 \pmod{1984}$ .

**Abgabe bis Freitag, 12.11.2010, 12:00 Uhr**