Übungen zu Vertiefung Elementare Zahlentheorie

WS 2010/2011, Blatt 6

Aufgabe 21. (a) Determine the remainders of 5^6, 5^8 and 1945^8 when devided by 7.

(b) Determine the remainders of 5^{10}, 5^{12} and 1945^{12} when devided by 11.

(c) Determine the remainders of 314^{162} when devided by 163, 7 and 165.

Aufgabe 22. (a) Determine the remainders of 2001^{2001} and 2011^{2011} when devided by 77.

(b) Determine the last digit in the decimal representation of 2007^{2007} and of 4567^{123}.

Aufgabe 23. Show:

(a) If p is a prime number, then

$$1^{p-1} + 2^{p-1} + \cdots + (p-2)^{p-1} + (p-1)^{p-1} \equiv -1 \pmod{p}.$$

(b) If p is a prime number $\neq 2$, then

$$1^p + 2^p + \cdots + (p-2)^p + (p-1)^p \equiv 0 \pmod{p}.$$

Aufgabe 24. Show: If p is a prime number $\neq 2$, then

$$1^2 \cdot 3^2 \cdots (p-4)^2 \cdot (p-2)^2 \equiv (-1)^{(p+1)/2} \pmod{p}$$

and

$$2^2 \cdot 4^2 \cdots (p-3)^2 \cdot (p-1)^2 \equiv (-1)^{(p+1)/2} \pmod{p}.$$

Abgabe bis Freitag, 26.11.2010, 12:00 Uhr