

Übungen zu Vertiefung Elementare Zahlentheorie

WS 2010/2011, Blatt 10

Aufgabe 37. (a) Prove: If p is a prime divisor of $2^{2^n} + 1$ ($n \geq 1$), then 2^{n+1} is a divisor of $p - 1$ (determine the order of 2 modulo p).

(b) Find the smallest prime divisor of $2^{32} + 1$.

Aufgabe 38. Determine for $p = 23$ and for $p = 31$ all integers a with $1 \leq a \leq p - 1$ that are quadratic residues modulo p .

Aufgabe 39. Calculate the following Legendre symbols:

(a) $\left(\frac{33}{71}\right), \left(\frac{34}{71}\right), \left(\frac{35}{71}\right), \left(\frac{36}{71}\right);$

(b) $\left(\frac{1234}{4567}\right), \left(\frac{4321}{4567}\right).$

Aufgabe 40. Determine all prime numbers $p \neq 3$ (resp. $p \neq 5$ resp. $p \neq 2, 5$) such that 3 (resp. 5 resp. 10) is a quadratic residue modulo p .

Abgabe bis DONNERSTAG, 23.12.2010, 12:00 Uhr