Vertiefung Elementare Zahlentheorie

WS 2010/2011, Wiederholungsblatt 2

Die folgenden Aufgaben sollen nur zur Selbstkontrolle dienen; Lösungen müssen nicht abgegeben werden.

Aufgabe 31. Give the definitions of a pythagorean triple and a primitive pythagorean triple.

Aufgabe 32. Describe all primitive pythagorean triples.

Aufgabe 33. Calculate all primitive pythagorean triples (x, y, z) with $20 < z < 30$.

Aufgabe 34. Let (x, y, z) be a primitive pythagorean triple. Show: Exactly one of the numbers x and y ist even; z is odd; if x is even, then x is divisible by 4.

Aufgabe 35. Let (x, y, z) be a primitive pythagorean triple. Show: Exactly one of the numbers x and y is divisible by 3; z is not divisible by 3.

Aufgabe 36. Let (x, y, z) be a primitive pythagorean triple. Show: Exactly one of the numbers x, y and z is divisible by 5.

Aufgabe 37. Show: If a and b are relatively prime integers > 0 such that ab is a square, then a and b are squares.

Aufgabe 38. State the two-squares theorem.

Aufgabe 39. Which of the following integers are sums of two squares?
$1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111$

Aufgabe 40. Express all primes < 50, as far as possible, as sums of two squares.

Aufgabe 41. Express $260 = 2^2 \cdot 5 \cdot 13$, $986 = 2 \cdot 17 \cdot 29$ and $1517 = 37 \cdot 41$ as sums of two squares.

Aufgabe 42. Let a and b be integers. Show: If p is an odd prime divisor of $a^2 + b^2$, but not a common divisor of a and b, then $p \equiv 1 \pmod 4$.

Aufgabe 43. State the three-squares theorem.

Aufgabe 44. Show directly (i.e. without using the three-squares theorem): Any integer of the form $8k + 7$ with $k \geq 0$ is not a sum of three squares.

Aufgabe 45. State the four-squares theorem.