Algebra II
13. Übungsblatt

William Crawley-Boevey

Abgabe: Bis zum 12.07.24 um 10:00h im Postfach Ihres Tutors
[Sarah Meier: 129]

Aufgabe 13.1. (2+2) Let $G = D_4$, the dihedral group of symmetries of a square. We have

$$G = \{1, \sigma, \sigma^2, \sigma^3, \tau, \tau\sigma, \tau\sigma^2, \tau\sigma^3\}$$

where σ is a rotation by angle $\pi/2$ and τ is a reflection. The conjugacy classes are

$$1, \{\sigma, \sigma^{-1}\}, \{\sigma^2\}, \{\tau, \tau\sigma^2\}, \{\tau\sigma, \tau\sigma^3\}.$$

(i) Let $N = \{1, \sigma^2\}$, a normal subgroup of G. Using the isomorphism between G/N and Klein's four group V, find four characters of degree 1 of G.

(ii) By decomposing the regular representation (or otherwise) find the fifth irreducible character of G.

Aufgabe 13.2. (4) Let $\pi \in S_n$ be a permutation whose decomposition as a product of disjoint cycles involves a_i cycles of length i for each $1 \leq i \leq n$. Show that the conjugacy class of π has

$$\frac{n!}{1^{a_1} 2^{a_2} \dots a_1! a_2! \dots}$$

elements.

Aufgabe 13.3. (1+1+1+1) Let $G = S_5$, the symmetric group of degree 5. Let $\mathbb{C}$ be the trivial representation, S the sign representation and V the natural permutation representation. Their characters χ_1, χ_2 and ϕ are as follows:

g_j	1	(12)	(12)(34)	(123)	(123)(45)	(1234)	(12345)
n_j	1	10	15	20	20	30	24
χ_1	1	1	1	1	1	1	1
χ_2	1	-1	1	1	-1	-1	1
ϕ	5	3	1	2	0	1	0

(i) Show that $\psi = \phi - \chi_1$ is a character, and that it is irreducible.
(ii) Find the characters ψ_s and ψ_a corresponding to $T^2(W)_s$ and $T^2(W)_a$, where W is the representation corresponding to ψ. (Use Aufgabe 12.4.)
(iii) Show that ψ_a is an irreducible character.
(iv) Show that $\xi = \psi_s - \chi_1 - \psi$ is a character, and that it is irreducible.

Aufgabe 13.4. (4) Complete the character table of S_5.