18th NRW Topology Meeting – Bielefeld (Germany)

Saturday, November 17, 2012 9.30, Hörsaal 5

Lennart Meier: (Bonn) "Galois Theory in Derived Algebraic Geometry"

Derived algebraic geometry is an analogue of classical algebraic geometry, replacing rings by homotopical objects like ring spectra. One important example in the field is the moduli stack of elliptic curves, upon which Goerss, Hopkins and Miller constructed a sheaf of commutative ring spectra \mathcal{O}^{top} . Its global sections are TMF, the spectrum of topological modular forms.

We will show that the ∞ -categories of quasi-coherent \mathcal{O}^{top} -modules and of TMF-modules are equivalent. This is a genuine phenomenon of *derived* algebraic geometry; analogous statements in classical algebraic geometry can hold only for affine schemes. Our proof exhibits a class of Galois extensions of TMF (in the sense of Rognes) and then uses Galois descent.

If one tries to generalize, one sees that a central point in the proof is the existence of a finite complex C (for every prime p) such that $TMF_{(p)} \wedge C$ is Landweber exact. We will present techniques towards the goal to prove analogous statements for other spectral stacks.